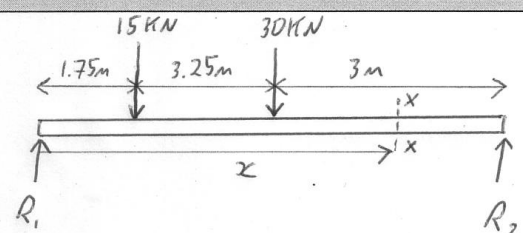




EXAMINATION SOLUTIONS: SESSION 2008/2009

School:	LOGISTICS & MANUFACTURING ENGINEERING		
Programme:	BENG MANUFACTURING SYSTEMS ENG.	Year:	2
Module/Unit:	STRESS ANALYSIS & DYNAMICS		
Author:	KELVIN LAKE	Page:	1 of 16

Question No	Question	Marks
① a	 $\sum M R_2 = 0$ $8R_1 = 15 \times 6.25 + 30 \times 3$ $8R_1 = 183.75$ $R_1 = 22.97 \text{ kN}$ $EI \frac{d^2 y}{dx^2} = M$ $= 22970x - 15000(x-1.75) - 30000(x-5)$ $\therefore EI \frac{dy}{dx} = \frac{22970x^2}{2} - \frac{15,000(x-1.75)^2}{2} - \frac{30,000(x-5)^2}{2} + A *$ $\therefore EI y = \frac{22970x^3}{6} - \frac{15,000(x-1.75)^3}{6} - \frac{30,000(x-5)^3}{6} + Ax + B **$	② ① ① ① ①

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	2 of 16
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Question No	Question	Marks
①a CONT	BOUNDARY CONDITIONS WHEN $x=0, y=0$ $\therefore$ FROM ** $B=0$ WHEN $x=8, y=0$ $\therefore$ FROM ** $0 = \frac{22970x^3}{6} - \frac{15,000(x-1.75)^3}{6} - \frac{30,000(x-5)^3}{6} + Ax$ $0 = \frac{22970 \times 8^3}{6} - \frac{15,000(6.25)^3}{6} - \frac{30,000 \times 3^3}{6} + 8A$ $0 = 1960106.7 - 610351.6 - 135,000 + 8A$ $\therefore 8A = -1214755$ $\therefore A = \underline{\underline{-151,844}}$ FROM ** $EIy = \frac{22970x^3}{6} - \frac{15,000(x-1.75)^3}{6} - \frac{30,000(x-5)^3}{6} - 151,844x$ AT MIDPOINT $x=4$ $EIy = \frac{22970 \times 4^3}{6} - \frac{15,000 \times 2.25^3}{6} - 151,844 \times 4$ $= 245013 - 28477 - 607376$ $= -390,840$	① ① ② ①

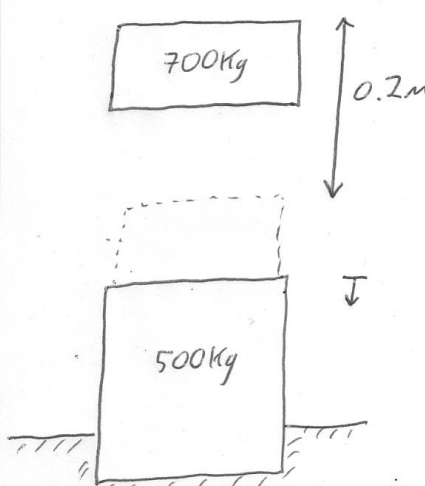
Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	3 of 16
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Question No	Question	Marks
	IF $y = -7.5 \text{ mm}$ $EI = \frac{-390,840}{-7.5 \times 10^{-3}}$ $= \underline{52.1 \text{ MPa}}$	(3)
(b)	$EI = 52.1 \text{ MPa}$ $= 52.1 \times 10^6$ $\therefore I = \frac{52.1 \times 10^6}{200 \times 10^9}$ $= \underline{2.605 \times 10^{-4} \text{ m}^4}$	(2)
(c)	$I = 2.605 \times 10^{-4} \text{ m}^4$ $= (2.605 \times 10^{-4}) \times 10^8$ $= \underline{26,050 \text{ cm}^4}$ LIGHTEST BEAM IS $D \times B = 457.2 \times 151.9$ $I_{xx} = 28,577 \text{ cm}^4$ WEIGHT = 67 kg/m $\therefore$ WEIGHT OF BEAM IS $8 \times 67 = \underline{536 \text{ kg}}$	(1) (1)

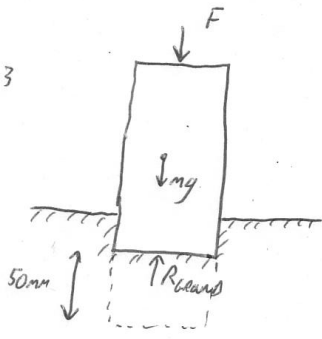
Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	4 of 16
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Question No	Question	Marks
(2) (a)	 700Kg 0.2m 500Kg ↓ $v = \text{VELOCITY OF DRIVER IMMEDIATELY BEFORE IMPACT}$ $v^2 = u^2 + 2as$ $= 0^2 + 2 \times 9.81 \times 0.2$ $= 3.924$ $\therefore v = 1.98 \text{ m/s}$ $u = \text{COMMON VELOCITY IMMEDIATELY AFTER IMPACT}$ CONSERVATION OF MOMENTUM $m_D v_D + m_P v_P = m_D u_D + m_P u_P$ $700 \times 1.98 + 500 \times 0 = 700 u_D + 500 u_P$ $1386 = 1200 u_D \quad (\text{SINCE } u_D = u_P)$ $\therefore u_D = 1.155 \text{ m/s}$	(1) (2)

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	5 of 16
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Question No	Question	Marks
(i)	BEFORE IMPACT $K.E. = \frac{1}{2} M_D V^2$ $= \frac{1}{2} \times 700 \times 1.98^2$ $= \underline{\underline{1372.1 \text{ J}}}$ AFTER IMPACT $K.E. = K.E._{\text{DOWNER}} + K.E._{\text{PILE}}$ $= \frac{1}{2} M_D u^2 + \frac{1}{2} M_P u^2$ $= \frac{1}{2} \times 700 \times 1.155^2 + \frac{1}{2} \times 500 \times 1.155^2$ $= \underline{\underline{800.4 \text{ J}}}$ $\therefore \Delta K.E. = 1372.1 - 800.4 = \underline{\underline{571.7 \text{ J}}}$ LOSS DUE TO HEAT, NOISE, DISTORTION. ii- REACTION IS CONSTANT THEN ACCELERATION MUST BE CONSTANT SINCE $F = ma$ $v^2 = u^2 + 2as$ $0 = 1.155^2 + 2a \times 50 \times 10^{-3}$ $\therefore a = \underline{\underline{-13.34 \text{ ms}^{-2}}}$ 	(1) (1) (1) (2)

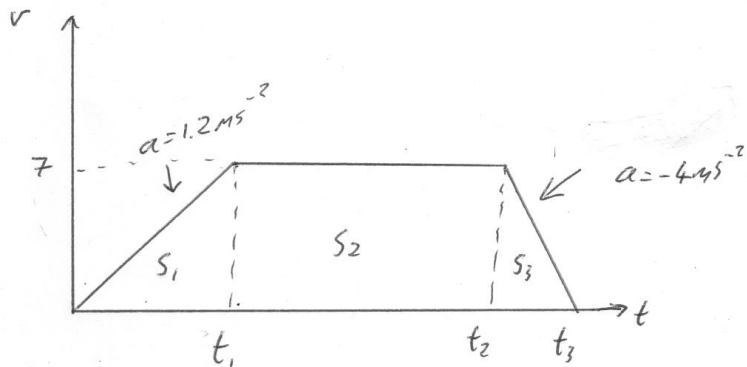
Continuation Sheet

Module/Unit:	STATICS ANALYSIS & DYNAMICS	Page:	6 of 16
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Question No	Question	Marks
(b)	now $R = F + mg$ $= (M_0 + M_p)a + (M_0 + M_p)g$ $= (700 + 500)13.34 + (700 + 500) \times 9.81$ $= 1200 \times 13.34 + 1200 \times 9.81$ $= \underline{\underline{27,780 \text{ N}}}$ RESISTANCE IS LIKELY TO CHANGE AS THE GROUND IS COMPACTED. (b) $m = 205 \text{ kg}$ $r = 0.45 \text{ m}$ RADIUS OF CYLINDER = 0.4 m 7 ms^{-1} ↑ $m = 1.5 \text{ kg/m}$ 300kg ACCELERATES AT 1.2 ms^{-2} DECELERATES AT 4 ms^{-2}	(2)

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	7 of 16
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Question No	Question	Marks
(i)	 $S_{TOTAL} = 80m$ <u>WITH ACCELERATION</u> $u=0, v=7, a=1.2, t=?, s=?$ $v = u + at$ $v^2 = u^2 + 2as$ $t = \frac{v-u}{a}$ $s = \frac{v^2 - u^2}{2a}$ $= \frac{7-0}{1.2}$ $= \frac{7^2 - 0^2}{2 \times 1.2}$ $= \underline{\underline{5.83s}}$ $= \underline{\underline{29.4m}}$ <u>WITH ACCELERATION</u> $u=7, v=0, a=-4, t=?, s=?$ $t = \frac{v-u}{a}$ $s = \frac{v^2 - u^2}{2a}$ $= \frac{0-7}{-4}$ $= \frac{0^2 - 7^2}{2 \times -4}$ $= \underline{\underline{1.75s}}$ $= \underline{\underline{6.125m}}$	(1) (1) (1)

Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	8 of 16
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Question No	Question	Marks
(i)	$S_{TOTAL} = S_1 + S_2 + S_3$ $80 = 29.4 + S_2 + 6.125$ $\therefore S_2 = \underline{\underline{44.5m}}$ CONSTANT VELOCITY $v = \frac{d}{t} \therefore t = \frac{d}{v} = \frac{44.5}{7} = \underline{\underline{6.35s}}$ $\therefore TOTAL TIME = 5.83 + 1.75 + 6.35$ $= \underline{\underline{13.9s}}$	(2)
(b) ii	ROPE TENSION AT DRUM = $M_{TOT} \times a_{TOT}$ $= (M_{CAGE} + M_{ROPE}) \times (a + g)$ $= (300 + 80 \times 1.5) \times (1.2 + 9.81)$ $= \underline{\underline{4624.2N}}$ $\therefore$ TORQUE TO ACCELERATE CAGE + DRUM = 4624.2×0.45 $= \underline{\underline{2080.9Nm}}$ HOWEVER TORQUE IS ALSO NEEDED TO ROTATE DRUM ITSELF $T_{DRUM} = I_{DRUM} \alpha_{DRUM}$ NOW $\alpha_{DRUM} = \frac{a}{r} = \frac{1.2}{0.45} = \underline{\underline{2.67 rad s^{-2}}}$	(2)

Continuation Sheet

Module/Unit:	STATS ANALYSIS & DYNAMICS	Page:	9 of 16
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Question No	Question	Marks
	$I_{\text{new}} = m k^2$ $= 205 \times 0.4^2$ $= \underline{\underline{32.8 \text{ kgm}^2}}$ $\therefore T = I \alpha$ $= 32.8 \times 2.67$ $= \underline{\underline{87.6 \text{ Nm}}}$ $\therefore \text{TOTAL TORQUE} = 2080.9 + 87.6$ $= \underline{\underline{2168.5 \text{ Nm}}}$	(2) (1)

Continuation Sheet

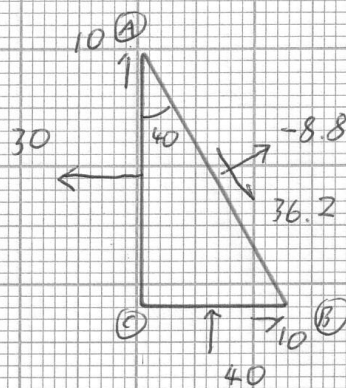
Module/Unit:	STATS ANALYSIS & DYNAMICS	Page:	10 of 16
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Question No	Question	Marks
(3)	$\theta = 90 - 50 = 40^\circ$ $\sigma_x = 30 \text{ MPa}$ $\sigma_y = -40 \text{ MPa}$ $\tau_{xy} = -10 \text{ MPa}$ $\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cos 2\theta + \tau_{xy} \sin 2\theta$ $= \frac{1}{2}(30 - 40) + \frac{1}{2}(30 + 40) \cos 80 - 10 \sin 80$ $= \underline{\underline{-8.8 \text{ MPa}}}$ $\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta - \tau_{xy} \cos 2\theta$ $= \frac{1}{2}(30 + 40) \sin 80 + 10 \cos 80$ $= \underline{\underline{36.2 \text{ MPa}}}$	(1)
(4)	$\sigma_1 = \frac{1}{2} \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \right]$ $= \frac{1}{2} \left[(30 - 40) + \sqrt{(30 + 40)^2 + 4(-10)^2} \right]$ $= \frac{1}{2} \left(-10 + \sqrt{4900 + 400} \right)$ $= \underline{\underline{31.4 \text{ MPa}}}$	(2)
		(2)

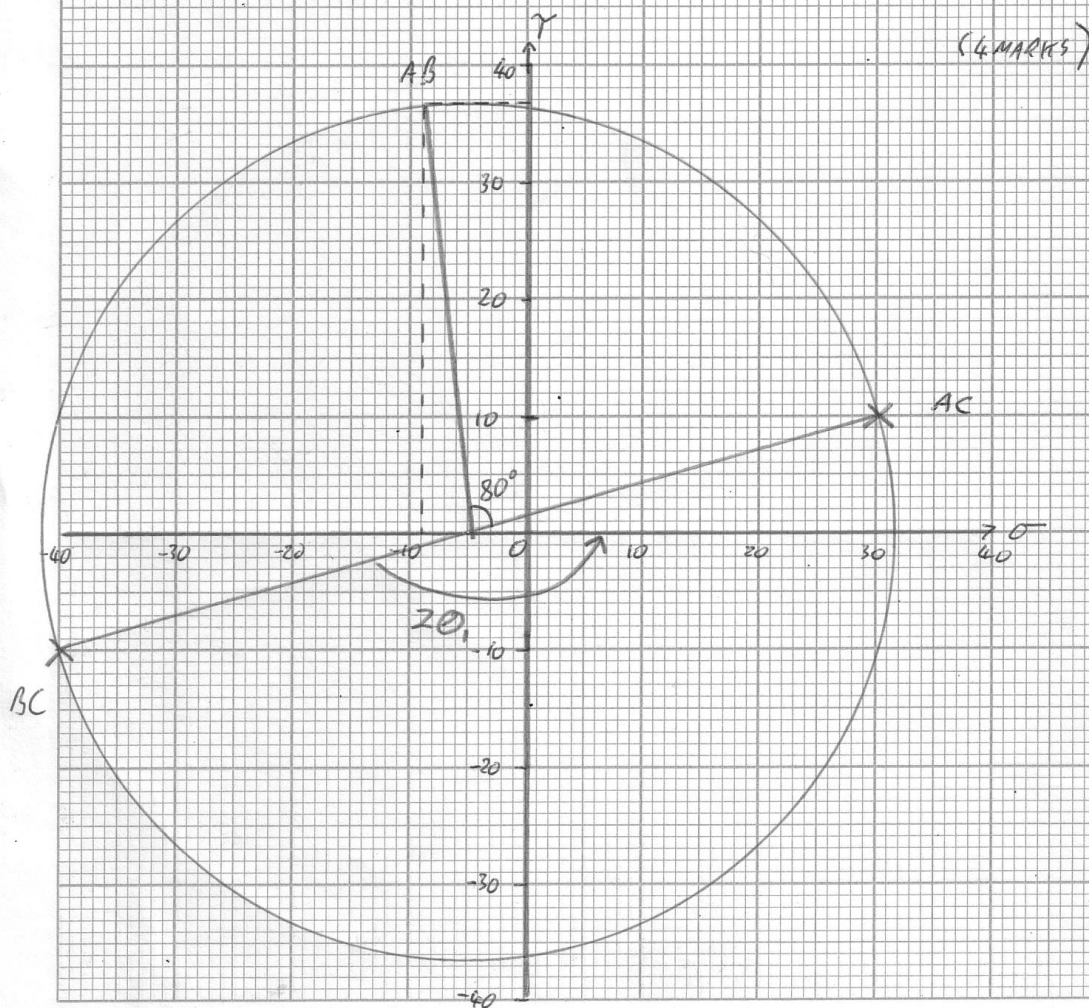
Continuation Sheet

Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	11 of 16
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Question No	Question	Marks
c)	$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$ $= \frac{1}{2} \sqrt{(30 + 40)^2 + 4(-10)^2}$ $= \underline{\underline{36.4 \text{ MPa}}}$	(2)
d)	SEE MOHR'S CIRCLE OVER PAGE. TAKING MEASUREMENTS $\sigma_0 \approx -9 \text{ MPa}$ $\tau_0 \approx 36.5 \text{ MPa}$ $\sigma_1 \approx 31.5 \text{ MPa}$ $\tau_{max} \approx 37 \text{ MPa}$	(4)
e)	BY MEASUREMENT $2\theta_1 = 165^\circ$ $\therefore \theta_1 = \underline{\underline{82.5^\circ}}$	(2)
f)	d) IS APPROXIMATE AS THEY ARE MEASURED	(1)



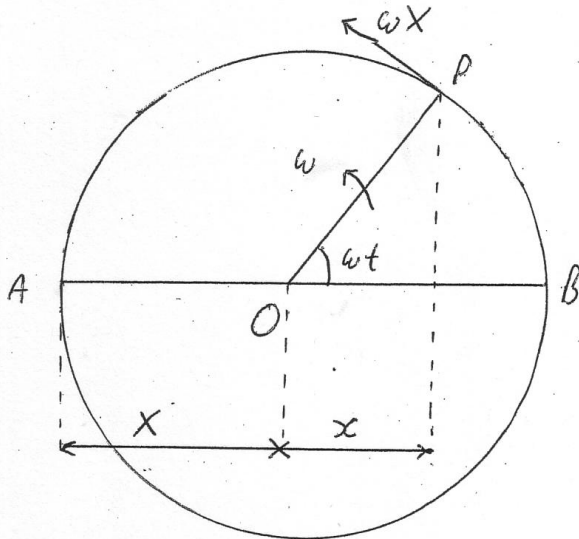
PLOT: AC : $\sigma_x = 30 \text{ MPa}$
 $\tau_{xy} = 10 \text{ MPa}$
 BC : $\sigma_y = -40 \text{ MPa}$
 $\tau_{xy} = -10 \text{ MPa}$



Continuation Sheet

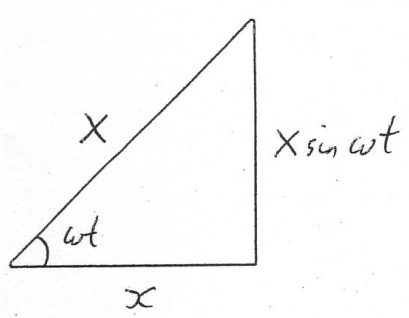
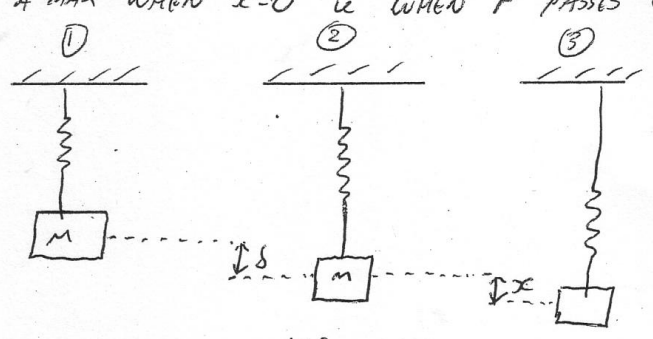
Module/Unit: STRESS ANALYSIS & DYNAMICS

Page: 13 of 16

Question No	Question	Marks
(2a)	 • DISTANCE x $x = X \cos \omega t$ • $\therefore$ VELOCITY $v = \omega X \sin \omega t$ (2)	1 1/2

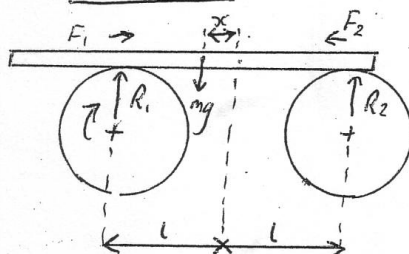
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Module/Unit:	STRESS ANALYSIS & DYNAMICS	Page:	14 of 16
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Question No	Question	Marks
(2a) COM.	now  $\therefore (X \sin \omega t)^2 + x^2 = X^2$ $\therefore \underline{X \sin \omega t = \sqrt{X^2 - x^2}} \quad (1)$ COMBINE (2) & (3) $\therefore \underline{v = \omega \sqrt{X^2 - x^2}}$ V IS A MAX WHEN $x=0$ i.e. WHEN P PASSES O  (2) IN EQUILIBRIUM $mg = k\delta$ (3) WHEN OSCILLATING RESTORING FORCE $F = kx$ • NEWTONS 2nd LAW $F = ma$ $\therefore ma = kx \therefore \underline{\underline{\frac{a}{x} = \frac{k}{m}}}$ *	(1/2) (2)
(2b)	(3)	

Continuation Sheet

Module/Unit:	SIXES ANALYSIS & DYNAMICS	Page:	15 of 16
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Question No	Question	Marks
2b CONT	now $f = \frac{1}{T} = \frac{1}{2\pi} \omega$ ALSO $a = \omega^2 x \therefore \omega = \sqrt{\frac{a}{x}}$ $\therefore f = \frac{1}{2\pi} \sqrt{\frac{a}{x}}$ $\therefore \text{FROM * } f = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$  TAKING MOMENTS ABOUT R_2 $mg(l+x) = R_1 \times 2l$ $\therefore R_1 = \frac{mg}{2l} (l+x)$ TAKING MOMENTS ABOUT R_1 $mg(l-x) = R_2 \times 2l$ $\therefore R_2 = \frac{mg}{2l} (l-x)$	① ① ② ②

Continuation Sheet

Module/Unit:	STATICS ANALYSIS & DYNAMICS	Page:	16 of 16
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Question No	Question	Marks
(2c) cont	now $F_1 = \mu R_1$ AND $F_2 = \mu R_2$ $= \frac{\mu mg}{2l} (1+x)$ $= \frac{\mu mg}{2l} (1-x)$ (2) $\therefore$ RESULTANT FORCE $F = F_1 - F_2$ $= \frac{\mu mg}{2l} (1+x) - \frac{\mu mg}{2l} (1-x)$ $= \frac{\mu mg}{2l} (1+x - (1-x))$ $F = \frac{\mu mgx}{l}$ $F = ma \therefore a = \frac{F}{m} = \frac{\mu gx}{l}$ (2) now $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \times \frac{1}{\omega}$ SINCE $a = \omega^2 x$, $\omega = \sqrt{\frac{a}{x}}$ $\therefore T = 2\pi \sqrt{\frac{\text{DISPLACEMENT}}{\text{ACCELERATION}}}$ $= 2\pi \sqrt{\frac{x l}{\mu g x}}$ $\therefore T = 2\pi \sqrt{\frac{l}{\mu g}}$ (2)	